# Induced representations of quantum kinematical algebras[1]

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**Abstract**

We construct the induced representations of the null-plane quantum Poincaré and quantum kappa Galilei algebras in $(1+1)$ dimensions. The induction procedure makes use of the concept of module and is based on the existence of a pair of Hopf algebras with a nondegenerate pairing and dual bases.

## 1 Introduction

Quantum kinematical algebras and groups are used for the study of $q$–deformed symmetries of the $q$–deformed space-time, which can be considered as a non-commutative homogeneous space of the quantum kinematical groups [1].

It is well known in nondeformed Lie group theory that given a Lie group $G$ and a closed Lie subgroup $K$ of it, the space of functions defined in the homogeneous space $(\simeq G/K)$ carries a representation of $G$ induced by a representation of $K$. Hence, symmetries and homogeneous spaces are closely related to induced representations. On the other hand, the physical interest of the induced representations are out of doubt [2, 3]. Thus, the study of the induced representations of quantum kinematical groups can be useful for determining the behaviour of physical systems endowed with deformed symmetries.

In this work we present the induced representations of the quantum $\tilde{\kappa}$–Galilei algebra and the null-plane quantum Poincaré algebra both in $(1+1)$ dimensions.

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The induction procedure used for us has a algebraic character since it makes use of the theory of modules which is, from our point of view, the appropriate tool to deal with the algebraic structures displayed by quantum groups and algebras [4, 5, 6, 7]. A similar method has been developed by Dobrev in [8, 9] and references therein. Both procedures deal with the dual case, closer to the classical one, constructing representations in the algebra sector. Also, one can find other papers extending the induction technique to the quantum case but constructing corepresentations of quantum groups, i.e. representations of the coalgebra sector, from a mathematical perspective [10, 11] as well as physical [13, 12, 14].

## 2 Induced representations of quantum groups

Let $H$ be a Hopf algebra and $V$ a linear vector space over a field $\mathbb{K}$ ($\mathbb{R}$ or $\mathbb{C}$). The triplet $(V, \triangleright, H)$ is said to be a left $H$–module if $\alpha$ is a left action of $H$ on $V$, i.e., a linear map $\alpha : H \otimes V \to V$ $(\alpha : (h \otimes v) \mapsto \alpha(h \otimes v) \equiv h \triangleright v)$ such that

$$h_1 \triangleright (h_2 \triangleright v) = (h_1 h_2) \triangleright v, \quad 1_H \triangleright v = v, \qquad \forall h_1, h_2 \in H, \ \forall v \in V.$$

Right $H$–modules can be defined in a similar way.

There are two canonical modules associated to any pair of Hopf algebras, $H, H'$ related by a nondegenerate pairing $\langle \cdot\,, \cdot \rangle$ (under these conditions $(H, H', \langle \cdot\,, \cdot \rangle)$ will be called a nondegenerate triplet):

1) The left regular module $(H, \succ, H)$ with action

$$h_1 \succ h_2 = h_1 h_2, \qquad \forall h_1, h_2 \in H.$$

2) The right coregular module $(H', \prec, H)$ with action defined by

$$\langle h_2, h' \prec h_1 \rangle = \langle h_1 \succ h_2, h' \rangle, \qquad \forall h_1, h_2 \in H, \quad \forall h' \in H',$$

that using the coproduct in $H'$ $(\Delta(h') = h'_{(1)} \otimes h'_{(2)})$ takes the form: $h' \prec h = \langle h, h'_{(1)} \rangle h'_{(2)}$.

The induction and coinduction algorithms of algebra representations are adapted to the Hopf algebras as follows. Let $(H, H', \langle \cdot\,, \cdot \rangle)$ be a nondegenerate triplet and $(V, \triangleright, K)$ a left $K$–module with $K$ a subalgebra of $H$. The carrier space, $\mathbb{K}^{\uparrow}$, of the coinduced representation is the subspace of $H' \otimes V$ with elements $f$ such that

$$\langle f, kh \rangle = k \triangleright \langle f, h \rangle, \qquad \forall k \in K, \ \forall h \in H. \tag{2.1}$$

The pairing used in expression (2.1) is $V$–valued and is defined by $\langle h' \otimes v, h \rangle = \langle h', h \rangle v$, where $h \in H, \ h' \in H', \ v \in V$. The action $h \triangleright f$ on the coinduced module is determined by

$$\langle h_1 \triangleright f, h_2 \rangle = \langle f, h_2 h_1 \rangle, \qquad \forall h_2 \in H.$$

Let $(\mathbb{K}, \triangleright, K)$ be a one-dimensional coinducing module. The carrier space of the coinduced representation is the subspace of $H' \otimes \mathbb{K} \simeq H'$ composed by elements $\varphi$ verifying the equivariance condition $\varphi \prec k = (1 \dashv k)\varphi, \ \forall k \in K$. The action of $H$ on $\mathbb{K}^{\uparrow}$ induced by the action of $K$ on $\mathbb{K}$ is given by

$$\langle h_2 \triangleright \varphi, h_1 \rangle = \langle \varphi, h_1 h_2 \rangle, \qquad \forall h_1, h_2 \in H, \quad \forall \varphi \in \mathbb{K}^{\uparrow},$$

or explicitly by $h \triangleright \varphi \equiv h \succ \varphi = \langle h, \varphi_{(2)} \rangle \varphi_{(1)}$.

It is worthy to note that to describe the induced module the right, $(H', \prec, H)$, and left, $(H', \succ, H)$, coregular modules are both pertinent, the former to determine the carrier space and the last to obtain the induced action.

Let us consider a nondegenerate triplet $(H, H', \langle \cdot \, , \cdot \rangle)$ with two finite sets of generators, $\{h_1, \ldots, h_n\}$ and $\{\varphi^1, \ldots, \varphi^n\}$, such that the families $\{h_l = h_1^{l_1} \cdots h_n^{l_n}\}_{l \in \mathbb{N}^n}$ and $\{\varphi^m = (\varphi^1)^{m_1} \cdots (\varphi^n)^{m_n}\}_{m \in \mathbb{N}^n}$ $(l = (l_1, \ldots, l_n), \ m = (m_1, \ldots, m_n))$ are bases of $H$ and $H'$, respectively. The action on the coregular module $(H', \succ, H)$ is obtained after to compute the action of the generators

$$h_i \succ \varphi^j = \sum_{k \in \mathbb{N}^n} \alpha_{ik}^j \varphi^k, \qquad i, j \in \{1, 2, \ldots, n\},$$

and to extend it to the ordered polynomial $\varphi^j = (\varphi^1)^{j_1} \cdots (\varphi^n)^{j_n}$ using the compatibility relation between the action and the algebra structure in $H'$

$$h \succ (\varphi\psi) = (h_{(1)} \succ \varphi)(h_{(2)} \succ \psi), \qquad h \succ 1_{H'} = \epsilon(h) 1_{H'}. \tag{2.2}$$

In order to write explicitly the expression of the action on a general ordered polynomial we take into account that:

1) There is a natural representation $\rho$, associated to $(H, \prec, H)$, of $H$

$$[\rho(h_2)](h_1) = h_1 \prec h_2.$$

2) The action on $(H', \succ, H)$ can be expressed in terms of $\rho$ using the adjoint with respect to $\langle \cdot \, , \cdot \rangle$ ($f^{\dagger} : H' \to H'$ is the adjoint of $f : H \to H$ if $\langle \, h, f^{\dagger}(h') \rangle = \langle \, f(h), h' \rangle$) defined by

$$h \succ \varphi = [\rho(h)]^{\dagger}(\varphi). \tag{2.3}$$

If the bases $\{h_l\}_{l \in \mathbb{N}^n}$ and $\{\varphi^m\}_{m \in \mathbb{N}^n}$ are dual, i.e. $\langle h_l, \varphi^m \rangle = l! \ \delta_l^m, \ \forall l, m \in \mathbb{N}^n$ (where $l! = \prod_{i=1}^n l_i!, \ \delta_l^m = \prod_{i=1}^n \delta_{l_i}^{m_i}$), we define "multiplication" operators $\overline{h}_i$, $\overline{\varphi}^j$ and

formal derivatives $\partial/\partial h_i$, $\partial/\partial\varphi^j$ by

$$\overline{h}_i(h_1^{l_1}\cdots h_i^{l_i}\cdots h_n^{l_n}) = h_1^{l_1}\cdots h_i^{l_i+1}\cdots h_n^{l_n},$$
$$\overline{\varphi}_i\left((\varphi^1)^{m_1}\cdots(\varphi^i)^{m_i}\cdots(\varphi^n)^{m_n}\right) = (\varphi^1)^{m_1}\cdots(\varphi^i)^{m_i+1}\cdots(\varphi^n)^{m_n},$$
$$\frac{\partial}{\partial h_i}(h_1^{l_1}\cdots h_i^{l_i}\cdots h_n^{l_n}) = l_i\; h_1^{l_1}\cdots h_i^{l_i-1}\cdots h_n^{l_n},$$
$$\frac{\partial}{\partial\varphi^i}\left((\varphi^1)^{m_1}\cdots(\varphi^i)^{m_i}\cdots(\varphi^n)^{m_n}\right) = m_i\;(\varphi^1)^{m_1}\cdots(\varphi^i)^{m_i-1}\cdots(\varphi^n)^{m_n}.$$

The adjoint operators are given by $\overline{h}_i^\dagger = \partial/\partial\varphi^i$ and $\overline{\varphi}^{i\dagger} = \partial/\partial h_i$.

## 3 Null–plane quantum Poincaré algebra

The null-plane quantum deformation of the $(1+1)$ Poincaré algebra, $U_z(\mathfrak{p}(1,1))$, is a $q$–deformed Hopf algebra that in a null-plane basis, $\{P_+, P_-, K\}$, has the form [15]

$$[K, P_+] = \tfrac{-1}{z}(e^{-2zP_+} - 1), \qquad [K, P_-] = -2P_-, \qquad [P_+, P_-] = 0;$$
$$\Delta P_+ = P_+\otimes 1 + 1\otimes P_+, \qquad \Delta X = X\otimes 1 + e^{-2zP_+}\otimes X, \quad X\in\{P_-, K\};$$
$$\epsilon(X) = 0, \quad X\in\{P_\pm, K\};$$
$$S(P_+) = -P_+, \qquad S(X) = -e^{2zP_+}X, \qquad X\in\{P_-, K\}.$$

It has also the structure of bicrossproduct $U_z(\mathfrak{p}(1,1)) = \mathcal{K}\triangleright\!\!\blacktriangleleft\mathcal{L}$, where $\mathcal{K}$ is a commutative and cocommutative algebra generated by $K$, and $\mathcal{L}$ is the commutative Hopf subalgebra of $U_z(\mathfrak{p}(1,1))$ generated by $P_+$ and $P_-$.

The dual Hopf algebra $F_z(P(1,1)) = \mathcal{K}^*\blacktriangleright\!\!\triangleleft\mathcal{L}^*$, where $\mathcal{K}^*$ is generated by $\varphi$, and $\mathcal{L}^*$ by $a_+$ and $a_-$, has the following structure

$$[a_+, a_-] = -2za_-, \qquad [a_+, \varphi] = 2z(e^{-\varphi} - 1), \qquad [a_-, \varphi] = 0;$$
$$\Delta a_\pm = a_\pm\otimes e^{\mp 2\varphi} + 1\otimes a_\pm \quad , \qquad \Delta\varphi = \varphi\otimes 1 + 1\otimes\varphi;$$
$$\epsilon(f) = 0, \quad f\in\{a_\pm, \varphi\}; \qquad S(a_\pm) = -a_\pm e^{\pm\varphi}, \qquad S(\varphi) = -\varphi.$$

The duality between $U_z(\mathfrak{p}(1,1))$ and $F_z(P(1,1))$ is explicitly given by the pairing

$$\langle K^m P_-^n P_+^p, \varphi^q a_-^r a_+^s\rangle = m!n!p!\;\delta_q^m\delta_r^n\delta_s^p.$$

## 3.1 Coregular modules

As we mentioned in the previous Section we need to know the left and the right coregular modules, $(F_z(P(1,1)), \succ, U_z(\mathfrak{p}(1,1)))$ and $(F_z(P(1,1)), \prec, U_z(\mathfrak{p}(1,1)))$ respectively, in order to construct the induced representations of $U_z(\mathfrak{p}(1,1)))$.

The structure of $(F_z(P(1,1)), \succ, U_z(\mathfrak{p}(1,1)))$ is given by

$$\begin{aligned} &K \succ (\varphi^q a_-^r a_+^s) = q\varphi^{q-1} a_-^r a_+^s + 2r\varphi^q a_-^r a_+^s + \tfrac{1}{z}\varphi^q a_-^r a_+[(a_+ - 2z)^s - a_+^s], \\ &P_- \succ (\varphi^q a_-^r a_+^s) = r\varphi^q a_-^{r-1} a_+^s, \qquad P_+ \succ (\varphi^q a_-^r a_+^s) = s\varphi^q a_-^r a_+^{s-1}. \end{aligned} \tag{3.1}$$

The following equalities are basic in the demonstration of the above result (3.1)

$$P_-^n K = K P_-^n + 2nP_-^n, \qquad P_+^n K = K P_+^n - n\frac{1}{z}(1 - e^{-2zP_+})P_+^{n-1}, \qquad \forall n \in N.$$

The structure of $(F_z(P(1,1)), \prec, U_z(\mathfrak{p}(1,1)))$ is given by

$$\begin{aligned} &(\varphi^q a_-^r a_+^s) \prec K = q\varphi^{q-1} a_-^r a_+^s, \qquad (\varphi^q a_-^r a_+^s) \prec P_- = r e^{2\varphi}\varphi^q a_-^{r-1} a_+^s, \\ &(\varphi^q a_-^r a_+^s) \prec P_+ = -\frac{1}{2z}\sum_{j=1}^{\infty}\sum_{k=0}^{j}\frac{1}{j}\begin{pmatrix} j \\ k \end{pmatrix}(-1)^k e^{-2\varphi}\varphi^q a_-^r (a_+ + 2kz)^s. \end{aligned} \tag{3.2}$$

The proof of (3.2) starts characterizing the module $(U_z(\mathfrak{p}(1,1)), \succ, U_z(\mathfrak{p}(1,1)))$. For that we take into account the following equalities

$$P_- K^n = (K+2)^n P_-, \qquad P_+ K^n = -\frac{1}{2z}\sum_{j=1}^{\infty}\frac{1}{j}(K - 2j)^n (1 - e^{2zP_+})^j, \qquad \forall n \in N,$$

that allow us to obtain easily the explicit expression of $(U_z(\mathfrak{p}(1,1)), \succ, U_z(\mathfrak{p}(1,1)))$

$$\begin{aligned} &K \succ K^m P_-^n P_+^p = K^{m+1} P_-^n P_+^p, \qquad P_- \succ K^m P_-^n P_+^p = (K+2)^m P_-^{n+1} P_+^p, \\ &P_+ \succ K^m P_-^n P_+^p = -\tfrac{1}{2z}\textstyle\sum_{j=1}^{\infty}\tfrac{1}{j}(K-2j)^m P_-^n (1 - e^{2zP_+})^j P_+^p. \end{aligned}$$

The corresponding endomorphisms of $U_z(\mathfrak{p}(1,1))$ are given by

$$\lambda(K) = \bar{K}, \qquad \lambda(P_-) = \bar{P}_- e^{2\frac{\partial}{\partial K}}, \qquad \lambda(P_+) = -\frac{1}{2z}\sum_{j=1}^{\infty}\frac{1}{j}e^{-2j\frac{\partial}{\partial K}}(1 - e^{2z\bar{P}_+})^j.$$

The computation of the adjoints gives

$$\begin{aligned} \lambda(K)^\dagger &= \tfrac{\partial}{\partial\varphi}, \qquad \lambda(P_-)^\dagger = e^{2\bar{\varphi}}\tfrac{\partial}{\partial a_-}, \\ \lambda(P_+)^\dagger &= -\tfrac{1}{2z}\textstyle\sum_{j=1}^{\infty}\tfrac{1}{j}(1 - e^{2z\frac{\partial}{\partial a_+}})^j e^{-2j\bar{\varphi}} = \tfrac{1}{2z}\ln\left[1 - e^{-2\bar{\varphi}}(1 - e^{2z\frac{\partial}{\partial a_+}})\right]. \end{aligned}$$

Hence, the action on $(F_z(P(1,1)), \prec, U_z(\mathfrak{p}(1,1)))$ is given by

$$f \prec K = \frac{\partial}{\partial\varphi} f, \quad f \prec P_- = e^{2\bar\varphi}\frac{\partial}{\partial a_-} f, \quad f \prec P_+ = \frac{1}{2z}\ln\left[1 - e^{-2\bar\varphi}(1 - e^{2z\frac{\partial}{\partial a_+}})\right] f. \quad (3.3)$$

The explicit action over the basis elements $\varphi^q a_-^r a_+^s$ (3.2) is obtained using the series expansions of the above expressions.

## 3.2 Induced representations

Let us consider the representation of $\mathcal{L}$

$$1 \dashv (P_-^n P_+^p) = \alpha_-^n \alpha_+^p, \qquad n, p \in \mathbb{N}, \quad \alpha_-, \alpha_+ \in \mathbb{C}. \quad (3.4)$$

The carrier space, $\mathbb{C}^\uparrow$, of the representation of $U_z(\mathfrak{p}(1,1))$, induced by the character (3.4), is constituted by the elements of $F_z(P(1,1))$ having the form

$$\phi(\varphi)e^{\alpha_- a_-}e^{\alpha_+ a_+}.$$

The induced representation can be translated to $\mathbb{C}[[\varphi]]$ where the action of the generators is

$$\phi(\varphi) \dashv K = \phi'(\varphi), \qquad \phi(\varphi) \dashv P_- = \phi(\varphi)\,\alpha_- e^{2\varphi},$$
$$\phi(\varphi) \dashv P_+ = \phi(\varphi)\tfrac{1}{2z}\ln[1 - e^{-2\varphi}(1 - e^{2z\alpha_+})].$$

A sketch of the construction of the representations induced by the character of $\mathcal{L}$ (3.4) is as follows [5]. To know the carrier space of the induced representation is characterized by the equivariance condition, which when is described in terms of the left regular module $(F_z(P(1,1)), \succ, U_z(\mathfrak{p}(1,1)))$ is reduced to the equations

$$\frac{\partial}{\partial a_-} f = \alpha_- f, \qquad \frac{\partial}{\partial a_+} f = \alpha_+ f.$$

which are not really differential equations, except at the limit $z \to 0$. However, their general solution is

$$f = \phi(\varphi)e^{\alpha_- a_-}e^{\alpha_+ a_+},$$

which is the same to that obtained working formally with the derivatives.

The right regular action (3.3) over $f$ gives the expression of the induced representation

$$\begin{aligned}
[\phi(\varphi)e^{\alpha_- a_-}e^{\alpha_+ a_+}] \prec K &= \phi'(\varphi)e^{\alpha_- a_-}e^{\alpha_+ a_+},\\
[\phi(\varphi)e^{\alpha_- a_-}e^{\alpha_+ a_+}] \prec P_- &= \phi(\varphi)e^{2\varphi}\alpha_- e^{\alpha_- a_-}e^{\alpha_+ a_+},\\
[\phi(\varphi)e^{\alpha_- a_-}e^{\alpha_+ a_+}] \prec P_+ &= \phi(\varphi)\tfrac{1}{2z}\ln\left[1 - e^{-2\varphi}(1 - e^{2z\alpha_+})\right] e^{\alpha_- a_-}e^{\alpha_+ a_+}.
\end{aligned}$$

Note that in reality we have two kinds of representations labeled by the pairs $(\alpha_+, 0)$ and $(\alpha_+, 1)$, respectively, since we can perform the rescaling $P_- \to P_-/\alpha_-$ and $a_- \to \alpha_- a_-$.

Let us consider now the character of $\mathcal{K}$

$$K^n \vdash 1 = c^n, \qquad n \in \mathbb{N}, \quad c \in \mathbb{C}. \tag{3.5}$$

We can construct a representation of $U_z(\mathfrak{p}(1,1))$ whose carrier space, $\mathbb{C}^\uparrow$, is formed by the elements of $F_z(P(1,1))$

$$e^{c\varphi}\phi(a_-, a_+).$$

The action on $\mathbb{C}^\uparrow$ can be carried to the subalgebra $\mathcal{L}^*$ of $F_z(P(1,1))$ obtaining

$$\begin{aligned} K \vdash f(a_-, a_+) &= [c + 2\bar{a}_- \tfrac{\partial}{\partial a_-} + \tfrac{1}{z}\bar{a}_+ (e^{-2z\frac{\partial}{\partial a_+}} - 1)] f(a_-, a_+), \\ P_\pm \vdash f(a_-, a_+) &= \tfrac{\partial}{\partial a_\pm} f(a_-, a_+). \end{aligned}$$

Effectively, the representation induced by the character of $\mathcal{K}$ (3.5) presents an equivariance condition described in terms of the left regular module by the equation $\partial f / \partial \varphi = cf$, whose general solution is

$$f = e^{c\varphi}\phi(a_-, a_+).$$

The restriction of the right regular action (3.1) over these elements gives the representation

$$\begin{aligned} K \succ [e^{c\varphi}\phi(a_-, a_+)] &= e^{c\varphi}\left[c - 2\bar{a}_- \tfrac{\partial}{\partial a_-} + \tfrac{1}{z}\bar{a}_+ (e^{-2z\frac{\partial}{\partial a_+}} - 1)\right] \phi(a_-, a_+), \\ P_\mp \succ [e^{c\varphi}\phi(a_-, a_+)] &= e^{c\varphi} \tfrac{\partial}{\partial a_\mp} \phi(a_-, a_+). \end{aligned}$$

This representation is called “local type” representation because when the deformation parameter goes to zero we recover the called local representations [3]. Note that the coefficient $c$ vanishes after the “gauge transformation” $K \to K - c$.

# 4 Quantum kappa Galilei algebra

The quantum algebra $U_{\tilde{\kappa}}(\mathfrak{g}(1,1))$, obtained by contraction of the $\kappa$–Poincaré [16], is characterized by the following algebraic structure [17]:

$$[H, K] = -P, \qquad [P, K] = \tfrac{1}{2\tilde{\kappa}} P^2, \qquad [H, P] = 0;$$

$$\Delta H = H \otimes 1 + 1 \otimes H, \qquad \Delta X = X \otimes 1 + e^{-\frac{1}{\tilde{\kappa}} H} \otimes X, \quad X \in \{P, K\};$$

$$\epsilon(X) = 0, \quad X \in \{H, P, K\};$$

$$S(H) = -H, \qquad S(X) = -e^{\frac{1}{\tilde{\kappa}} H} X, \quad X \in \{P, K\},$$

where $\tilde{\kappa} = \kappa c$, being $\kappa$ the deformation parameter of the above mentioned $\kappa$–Poincaré algebra.

The dual algebra $F_{\tilde{\kappa}}(G(1,1))$ is generated by $x, t$ and $v$, and its Hopf structure is

$$[t,x] = -\tfrac{1}{\tilde{\kappa}}x, \qquad [x,v] = \tfrac{1}{2\tilde{\kappa}}v^2, \qquad [t,v] = -\tfrac{1}{\tilde{\kappa}}v;$$
$$\Delta t = t\otimes 1 + 1\otimes t, \quad \Delta x = x\otimes 1 + 1\otimes x - t\otimes v, \quad \Delta v = v\otimes 1 + 1\otimes v;$$
$$\epsilon(f) = 0, \quad f\in\{v,t,x\};$$
$$S(v) = -v, \qquad S(x) = -x - tv, \qquad S(t) = -t.$$

The pairing between both Hopf algebras is given by

$$\langle K^m P^n H^p, v^q x^r t^s\rangle = m!n!p!\,\delta_q^m\delta_r^n\delta_s^p.$$

The action of $U_{\tilde{\kappa}}(\mathfrak{g}(1,1))$ on the left coregular module $(F_{\tilde{\kappa}}(G(1,1)), \succ, U_{\frac{1}{2\tilde{\kappa}}}(\mathfrak{g}(1,1)))$ is

$$K\succ f = \left[\frac{\partial}{\partial v} + \frac{1}{2\tilde{\kappa}}\bar{x}\frac{\partial^2}{\partial x^2} - \bar{t}\frac{\partial}{\partial x}\right] f, \qquad P\succ f = \frac{\partial}{\partial x}f, \qquad H\succ f = \frac{\partial}{\partial t}f,$$

where $f$ is an arbitrary element of $F_{\tilde{\kappa}}(G(1,1))$.

The action on the right coregular module $(F_{\tilde{\kappa}}(G(1,1)), \prec, U_{\tilde{\kappa}}(\mathfrak{g}(1,1)))$ is given by

$$f\prec K = \frac{\partial}{\partial v}f, \quad f\prec P = \frac{\frac{\partial}{\partial x}}{1-\frac{\bar{v}}{2\tilde{\kappa}}\frac{\partial}{\partial x}}f, \quad f\prec H = \left[\frac{\partial}{\partial t} - 2\tilde{\kappa}\ln(1-\frac{\bar{v}}{2\tilde{\kappa}}\frac{\partial}{\partial x})\right] f.$$

Now we can obtain a family of representations of $U_{\tilde{\kappa}}(\mathfrak{g}(1,1))$ coinduced by the character

$$1\dashv P^n H^p = a^n b^p, \qquad n,p\in\mathbb{N}, \quad a,b\in\mathbb{C},$$

of the abelian subalgebra of $U_{\tilde{\kappa}}(\mathfrak{g}(1,1))$ generated by $H$ and $P$, whose carrier space $\mathbb{C}^{\uparrow}$ is the set of elements of $F_{\tilde{\kappa}}(G(1,1))$ of the form [6, 4]

$$\phi(v)e^{ax}e^{bt}.$$

The action on $\mathbb{C}^{\uparrow}$ can be translated to the space of formal power series

$$\begin{aligned} \phi(v)\dashv K &= \phi'(v), \qquad \phi(v)\dashv P = \phi(v)\tfrac{a}{1-\frac{1}{2\tilde{\kappa}}av}, \\ \phi(v)\dashv H &= \phi(v)[b + 2\tilde{\kappa}\ln(1-\tfrac{1}{2\tilde{\kappa}}av)]. \end{aligned}$$

The gauge transformation $H\to H-b$ allows the "gauge equivalence" of the representations labeled by the pair $(a,b)$ and those parameterized by $(a,0)$.

The "local" representation of $U_{\tilde{\kappa}}(\mathfrak{g}(1,1))$ coinduced by the character of the abelian subalgebra of $U_{\tilde{\kappa}}(\mathfrak{g}(1,1))$ generated by $K$

$$K^m \vdash 1 = c^m, \qquad m \in \mathbb{N}, \quad c \in \mathbb{C},$$

has as support the subspace of $F_{\tilde{\kappa}}(G(1,1))$ of elements

$$e^{cv}\phi(x,t).$$

The action of $U_{\tilde{\kappa}}(\mathfrak{g}(1,1))$ carried to the subalgebra of formal power series $\mathbb{C}[[t,x]]$ is

$$\begin{aligned} K \vdash \phi(x,t) &= (c - \bar{t}\tfrac{\partial}{\partial x} + \tfrac{1}{2\tilde{\kappa}}\bar{x}\tfrac{\partial^2}{\partial x^2})\phi(x,t), \\ P \vdash \phi(x,t) &= \tfrac{\partial}{\partial x}\phi(x,t), \qquad H \vdash \phi(x,t) = \tfrac{\partial}{\partial t}\phi(x,t). \end{aligned}$$

Also here, the label $c$ can be reduced to zero.

Note that in the limit of the deformation parameter goes to zero we recover the well know induced representations of the corresponding nondeformed Lie groups.

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